1. Find the smallest positive integer n with the following property: there does not exist an arithmetic progression of 1999 real numbers containing exactly n integers.

2. Compute the sum $S = \sum_{i=0}^{101} \frac{x_i^3}{1-3x_i+3x_i^2}$ for $x_i = \frac{i}{101}$.

3. For a positive integer n let $S(n)$ be the sum of the digits in the decimal representation of n. Any positive integer obtained by removing several (at least one) digits from the right-hand end of the decimal representation of n is called a *stump* of N. Let $T(n)$ be the sum of all stumps of n. Prove that $n = S(n) + 9T(n)$.

4. Show that for any positive integers a and b, $(36a + b)(a + 36b)$ cannot be a power of 2.

5. Find all pairs of positive integers a and b such that

$$\frac{a^2+b}{b^2-a} \text{ and } \frac{b^2+a}{a^2-b}$$

are both integers.

6. Suppose $ABCD$ is a square piece of cardboard with side length A. On a plane are two parallel lines ℓ_1 and ℓ_2 which are also a units apart. The square $ABCD$ is placed on the plane so that sides AB and AD intersect ℓ_1 and ℓ_2 at E and F respectively. Also, sides CB and CD intersect ℓ_2 at G and H respectively. Let the perimeters of $\triangle AEF$ and $\triangle CGH$ be m_1 and m_2 respecetively. Prove that no matter how the square was placed, $m_1 + m_2$ remains constant.

7. Prove that there exists a triangle that can be cut into 2005 congruent triangles.

8. Given two positive integers m and n, find the smallest positive integer k such that among any k people, either there are $2m$ of them who form m pairs of mutually acquainted people, or there are $2n$ of the forming n pairs of mutually unacquainted people.